1 ibration Point Trajectory Design

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Orbits about the Sun-Earth L 1 and 12 libration points have become very popular for space physics *and astrophysics missions. Due to the hyperbolicity* of *the region of phase space where these orbits are found, they are difficult to compute and analyze. 7he invariant manifold structure provided by dynamical systems theory have been useful to compute transfer trajectories between orbits. These methods are very promising and require further development. This systematic approach is a great improvement from the difficult and /abor-intensive numerical search methods current/y popular in the astrodynamics community for studying these orbits. 1 he geometric constraints and mission critical issues are discussed to give the numerical dynamical systems community some insight into the practical considerations and important problems of interest to* the space mission designers. *It is hoped that this communication will lead to mom fruitful exchanges between the two communities.*

1. The Three Body Problem and Space Missions

1 he Three Body Problem has intrigued some the greatest minds of the last three centuries and has stimulated the development of science as few problems have. I-he crowning achievement is the qualitative method introduced by Poincare from which moden dynamical systems theory is derived. The advent of the computer age brought in a new revolution whereby previously undreamed-of numerical experiments can be performed by the stroke of a few keys. 1 his blurred the distinction between quantitative and qualitative methods in the following sense. Qaulitatiive methods not only provide a global understanding of the dynamics but now serve as essential guides in the numerical computations of dynamical quantities.

Parallel with the development of computers, space exploration also took its first steps in the past 50 years. Suddenly, the formerly important but academic three body problem is discovered to have practical significance for space missions. On August 12, 1978, the International Sun-Earth Explorer-3 (ISEE 3) was launched toward the interior Sun-Earth libration point, L1, and inserted into a "halo orbit" around L 1 under the direction of Dr. Robert W. Farquhar. Now periodic orbits and quasiperiodic orbits of the three body problem have taken on a completely new significance. They have become "practical and usefule". Along with utility comes the engineering issues and all the complexities of actually flying such a mission.

However, there is currently a gap between the astrodynamics and dynamical systems communities. The purpose of this paper is to narrow that gap by introducing the problems and methods to both communities. 1 hereby it is hoped that an exchange of ideas between the two would follow resulting in collaborations and new developments in the theory and applications of the three body problem to future space missions. While quasiperiodic orbits about the libration points are extremely useful for current space missions, their significance for the exploration of the solar system has only been glimpsed.

2. Orbits Around L1 and 1.2

For the purposes of this discussion, it suffices to use the circular restricted three body problem (RTBP). In this model, the two primary bodies are moving in circular orbits about their common center of mass and the infintessimal third body moves uder the gravitation of the primaries without exerting any gravitational force on the primaries in return. Fig. 1 depicts the Sun-Earth-Spacecraft system in rotating coordinates with the origin is at the center of mass; the X-axis containing the two primaries with the Sun to the left and the Earth to the right, the Z-axis is normal to the Ecliptic, the orbit plane of the primaries, and the Y-axis completes a right handed *coor-*dinate system. Euler discovered three collinear equilibrium solutions on the X-axis, labelled L1 to 13. Largrange discoverd the two equilateral triangular equilibrium solutions, labelled L4 and L5. The labelling convention used here is the most prevalent in the astrodynamics Community, although by no means standrad. The equilibrium points are collectively known as the libration points or the Lagrangian points.

The equations of motion is as follows:

x " - 2y ' = ux
y " - 2 x ' = Uy
z " = Uz

where

u = 1/2 (x^2 + y^2) + (1 -m)/d + m/r
m = normalized mass parameter of the Earth-Sun E/(S+E),
d = distance of the infinitessimal mass to the first primary,
P1 = [(x+m)^2 + y^2 + z^2]^1/2
r = distance of the infinitessimal mass to the second primary,
P2 = [(x-1+m)^2 + y^2 + z^2]^1/2

l'he normalization sets the sum the mass of PI and P2 to 1, the distance between P1 and P2 to 1, and the angular rate of PI and P2 around one another to 1 (period 2 Pi). See Szebehely for the derivation.

L1, L2, and L3 are unstable equilibria, while 1.4 and L5 are stable equilibria for planetary mass ratios (see Szebehely, 5.5). We will limit our discussion to 11 and L 2 in this paper since the other libration points are currently not as useful for space missions.

Linear analysis about L1 and L2 indicates that there are two families of periodic solutions for the linearized system - planar periodic oribts and vertical simple harmonic motion:

x = -A Cos(a t + f),
y = k(a) A Sin (a t + f),
z = B Sin (b t + g),

Both the family of periodic orbits of the linearized system may be continued to periodic orbits in the RTBP. These "modes" may be combined to form quasiperiodic orbits about L1 and 1..2 known as lissajous orbits as their YZ-projection is the familiar lissajous pattern. When the Y-amplitude, k(a)A, reaches a certain size (about 645,000 krn for the Sun-Earth system), depending on the mass parameter, the frequencies a and b become commensurate thereby producing a periodic orbit known as a halo orbit. Note that this is very far from the libration point and well beyond the region of validity for the linear expansion. This periodic orbit comes about from higher order nonlinear effects through a relation between the amplitudes A and B. Thus for halo orbits, there is effectively only one amplitude. 1 here are also large quasiperiodic orbits on tori around the halo orbits also known as lissajous orbits. In the Sun-Earth system, all of these orbits have periods of about 6 months. All of these orbits are unstable except for very large amplitudes where there is a narrow region of stability. For the Earth-Moon L1, for 73,000 km < B <74,5010 km, there is a band of stable halo orbits (Breakwell and Brown). Plots of the various projections of the lissjous orbits are given in Figures 2 and 3 at the end of the paper.

3. Why Libration Point Missions?

Why are libration point orbits useful to space missions? Why is there suddenly so much interest?

11 is in a sentinel position between the Earth and the Sun. "1 his is an ideal location for solar-terrestrial experiments such as the ISEE3, SOlar Heliospheric Observatory (SOH {0), Advanced Composition Explorer (ACE) and Suess-Urey Discovery (SU) missions (see list at end of section for brief description of missions). An orbit about L 1 provides continuous viewing of the Sun and the scalar wind outside the geomagnetosphere, a continuous link to the Earth, a constant thermal environment, and constant power for spacecraft through the solar panels. Similarly, an orbit about L? is ideal to study the geotail anti its interaction with the solar wind such as for the GEOTAIL mission. And there is an easy and inexpensive heteroclinic transfer orbit from an orbit about L 1 to an orbit about 12 requiring no more than 6 months for transfer. Figure 2 plots the three projections of the orbit for the SU mission currently being proposed 10 NASA. This is a large amplitude lissajous orbit. 1 he goal of SU is to collect and return samples of the ephemral solar wind particles in an orbit around L1. The orbital dynamics and mission design will be discussed in the next section.

The 12 region is ideal for astrophysics experiments such as the Far InfraRed Explorer Mission (FIRE) and Primordial Stuctures Investigation Mission (PSI). An orbit about L 2 provides a stable and cold environment perfectly suited for infrared and microwave telescopes. More than half of the celestial sphere is available at all times for observation so that in 6 months' time, a complete survey of the sky can be made. Similar to the L1 situation, the communications geometry, the power and thermal environment are nearly constant here. Figure 3 plots the three projections of an orbit for both the FIRE and PSI. 1 hese missions are cosmic microwave background (CMB) radiation experiments following in the footstep of the famous COBE Mission which determined the anisotropic clumping of matter in the universe. '1 he instruments to detect CMB radiation (around 3 deg Kelvin) need to stay very cold hence a small lissajous orbit around L 2 is ideal. In contrast, the COBE mission used an orbit around Earth. Note also that a lunar swingby was used to steer the spacecraft into orbit around 12 for PSI and FIRE.

Why is a constant environment so important? From the science point of view, it is highly desirable to have all observations made in a constant environment to reduce error sources and uncertainties which simplifies the data reduction, generally a massive task in itself. From the instrument and spacecraft point of view, a constant environment is less stressful on the hardware, much easier to design and trouble shoot should anomalies occur. From the operations point of view, a constant environment greatly simplifies planning activities both before and after the launch. Of course, this elegant solution ultimately translates to lower cost and greater savings in almost every aspect of the entire mission life cycle.

Compare with a typical low Earth mission with a peiord of one to two hours. Within every period, the Earth occults the Sun; this changes the temeprature and the resulting stress causes jitter in the instruments. While in shadow, batteries must supply the necessary power; this increases the spacecraft mass and complexity. Depending on the mission one may or may not want to see the Sun. Infrared telescopes typically require sub-kelvin coolers and the telescope boresight must always be 80 to 90 degrees away from the Sun and the Earth's limb. Add to this the constantly and quickly changing communcations geometry and you have less than 50% of the time where you can make observations per orbit. The constant slewing of the telescope to avoid the Sun and Earth, or pointing of the antenna for communcations all add to a highly labor intensive and complex mission operations which is more costly.

Another driver which favors libration point missions in the current environment is the ease with which the orbit can be achieved in comparison with other interplanetary missions. A direct launch from Earth to a halo orbit requires about 3 months. The transfer for a mission to Mars requires about 1 year, to Jupiter requires about 3 years, and to Pluto about 10 years. '1 he launch energy to achieve a halo orbit is less than that for a mission to orbit the Moon. The launch energy is measured by a quantity called C3 equal to twice the keplerian orbit energy. (The origin of the term C3 is unkown, but the constant of the vis-viva integral, Equation 19 of Section 87, in Moulton's "An Introduction to Celestial Mechanics" is called C3 anti is likely the source of this terminology.) For halo orbits, the C3 is -0.6 (km/sec)^2, the neagative sign indicating that it is still bound to the Earth. For a direct Mars transfer, the C3 is around 10; for Jupiter, the C3 is around 60, for Pluto, it's around 200. 1 bus, it takes less time and less launch capability to get to a halo orbit than fOr almost any other interplanetary mssion. '1 his translates to lower launch cost which is always a significant part of the overall mission cost.

F"or this discussion, all costs exclude the launch vehicle and arc in 1995 dollars. Typical mission costs have dropped from the billion dollar range to as low as $35 million in some cases A few of the libration point missions we have proposed are in the $60 to $100 million range. '1 hese factors, combined with the current detector technology which enabled some of these new mission concepts, have brought libration point trajectories to the forefront of space physics and astrophysics missions. Table 1 at the end of the paper lists the current and some of the proposed missions using libration point trajectories.

4. Trajectory Design Issues

In general, finding individual lissajous and halo orbits is not difficult. By exploiting the symmetry of halo orbits with respect to the xz-plane, one can easily find a halo orbit iteratively. Pick some point (x,0,z) near L1 or 1.2, pick an initial velocity (0,v,0), the form suggested by the symmetry, and integrate the equations of motion. Change v slightly until a periodic orbit is achieved. Both x and z may need to be moved about since these orbits are not dense in space. Using the variational equation, a newton scheme can quickly find these orbits provided a very good initial guess is given, Lissajous orbits do not have this symmetry property and are found by parallel shooting methods (Howell & Pernicka). 1 he initial guess is provided by a third order Linstedt-Poincare expansion for lissajous orbits (Richardson). SImo's group have computed these series to order 35 with various models providing extremely accurate solutions (Simo, Gomez, Llbre, and Martinez).

A Mission to L2

l-he challenge is in finding an orbit with useful properties, We examine some astrophysics missions at L2. The PSI and FIRE spacecrafts are designed to spin slowly around an axis pointed at the Sun (Lo, Howell, and Barden). The instruments are mounted perpendicular to this axis so the boresight is always 90 deg away from the Sun and the Earth and is never exposed to their radiation. As the spacecraft slowly spin about this axis, the instruments map out a great circle swath of the celestial sphere, mapping the entire sky in 6 months. The antennae and the solar panels are fixed at the' back of the spacecraft which is always pointed at the Sun. However, the antennae have a narrow beamwidth and the data rate is high enough that a small lissajous orbit must be used in order to guarantee the data rate without the need to steer the spacecraft from its Sun-pointed orientation (a fixed antenna is not pointable except by moving the entire spacecraft which is undesirable for these mission geometry constraints). 1 hus a small amplitude (120,000 km) lissajous orbit was selected. The amplitude is like the maximum radius of the orbit. But, the problem with small lissajous orbit is that it requires about 100 to 150 m/s of delta-V (change in velocity using thrusters and propellant) to insert into the lissajous orbit. Since the total propellant capability of the mission is about 200 m/s post-launch, this is significant. A lunar swingby is used to reduce this insertion delta-V to 15 m/s following the design for the Russian RELICT 2 Mission (Dunham).

A Mission to 1.1

We examine, next, a solar physics mission to 1.1. The SU Mission (Lo, Howell, and Barden) will spend two years in a large amplitude lissajous orbit about L1 to collect samples of the solar wind and then bring it back to earth. Unlike the astrophysics missions, it has very little data to downlink so the antenna pointing is not a problem. Consequently, it can use a large lissajous orbit (300,000 km Z amplitude, 700,000 km Y amplitude) which lowers the delta-V required to insert into the lissajous orbit to a negligible 5 m/s. (The ISEE3 halo orbit is 120,000 km by 666,000 km in the corresponding amplitudes.) However, the return segment to Earth is a challenge. Since the sample capsule will be captured by helicopters at a preselected site in Utah, the return must occur during the day time to facilitate the capture. Unfortunately, the dynamics of the problem favors a direct return on the night side. 10 force a direct return on the dayside is prohibitively expensive and well beyond the 300 m/s total delta-V capability of the SU spacecraft. The situation would be very different if it were returning from L2. However, we have observed heteroclinic behavior between orbits around 11 and L2. Using this observation, the spacecraft was sent to L2 after departing L1 to turn it around for Earth return on the day side

1 he Transfer Problem

The transfer problem is a significant problem in the trajectory design for a mission to L1 or 12. Even without the lunar swingby mentioned earlier, this is a difficult problem Farquhar and Dunham computed this trajectory directly from forward integration even in tile case of lunar swingbys. Another approach is to integrate backwards from a halo orbit. The instability of the dynamics makes it very easy to depart. However, the transfer trajectory must connect up with a typical 200 km circular launch orbit at 28.5 deg inclination. Thus timing becomes a problem because the real system is non-autonomous. If one shifts the time by some amount, the lissajous orbit is no longer "there". Another lissajous orbit must be computed to accommodate the new times. Needless to say, the entire process is complicated and labor intensive especially when a lunar encouter is required. The problem is highly nonlinear and it is extremely difficult to find an end-to-end trajectory satisfying all the constraints. 1 he optimization of the maneuvers along the trajectory further complicates the analysis. Consequently, it is virtually impossible to perform any in depth parametric studies.

1 his is a result of the lack of integrals so there are no orbital elements with which to parametrize the orbit design space. While it is true that missions with multiple planetary flybys such as the Galileo Mission are operating in the N-body regime, their energy is much higher "1 he flyby trajectories are hyperbolic orbits with large positive C3. Whereas the C3 for libration point missions is very close to O. '1 his low C3 is much closer in energy to thaf of the libration points. It is in this region of the phase space that chaotic phenomena and nonlinear effects are observed. 1 his is why conic approximations are completely unsuitable for this regime, but are excellent for planetary flybys. In order to understand and control the dynamics around I 1 and 12, we must turn to dynamical systems theory where great strides have been made in recent years both in theory and in applications to engineering problems.

5. The Need for Dynamical Systems Theory

The rich structures and methods available in modern dynamical systems theory have not been seriously considered by most astrodynamicists in the United States. The European Space Agency (Rodriguez-Canabal; Simo, Gomez, Llbre, and Martinez) and the Moscow Space Research Institute (Eliasberg, Timokhava, and Boyarski) in the 80's began studying the control of a spacecraft in halo orbits using invariant manifold theory. The basic idea is to project the spacecraft state onto the stable and unstable manifolds of the halo orbit and cancel out the unstable components with a maneuver ancf let the stable manifold bring the spacecraft back into a nearby halo orbit on the center manifold. This approach can greatly reduce the station keeping delta-V. For ISEE3 the actual stationkeeping was 8 r~u'sec/year, It was a propellant-rich mission with little need for efficiency. With the stable manifold control strategy, this can be reduced by several orders of magnitude to a few mm/see/year. But, it requires more frequent orbit determination and maneuvers which can be costly. Nevertheless, this strategy can be made practical arid has spurred the community to rethink the station keeping requirements.

Current estimates for stationkeeping near the libration points is about 4 m/s/year with maneuver frequency of once every 4 to 8 weeks using traditional methods. Whether it's 4 or 8 rids/year, this delta-V requirement is insignificant in comparison to maneuvers required for the launch error correction (requires > 100 m/s) or the halo orbit insertion (150 inks). Despite the elegance and originality of the stable manifold control, it has not been adopted by any mission to date. In the minds of the project managers, the benefits of this new approach is perhaps insufficient to warrant the changes required in the mission design and operations world to accommoate this new method.

However, what they did not realize is that this approach opened up an entire new vista for mission and trajectory design. Invariant manifold theory can be applied to many other aspects of trajectory design besides station keeping. It offers the possibility to parametrize the orbit space so that entire families of qusiperiodic orbits can be investigated. It offers a coherent theory describing hitherto inexplicable phenomena. It provides intuition and insight into the behavior of orbits that have been fuzzy and mysterious except to the few who have been numerically exploring this region of the phase space. Finally, this global approach is providing new handles from which new algorithms can be and have been developed to compute precise orbits with prescribed properties as required by the mission. Thus entire families of orbits can now be systematically computed in contrast to single orbits painstakingly conjured out of programs by tricks with slot of blood, sweat, and tear.

Here is an application of the invariant manifold theory we have used for some of our advanced mission concepts. We call this method the stable manifold transfer.'1 he launch problem has been greatly helped by an understanding of the stable manifolds of halo orbits. Since halo orbits arc periodic, using Floquet theory, their stable manifold can be quickly computed (Barden). By finding the point on the stable manifold closest to Earth, a candidate trajectory can be identified for a launch trajectory. Usually, the stable manifold does not approach the Earth as close as 200 km. However, having identified the most favorable trajectory on the stable manifold, we have also identified the most favorable insertion point. By adjusting a maneuver at the insertion point and the launch condtions, a launch trajectory can be quickly found In the case of the lunar transfer, one simply looks for where the stable manifold comes closest to the lunar orbit and adjust the timing accordingly to effect a lunar swingby. This was how the PSI and FIRE trajectoires were designed.

It has been observed by some, incorrectly, that the stable manifold method is just backwards integration. The stable manifold transfer algorithm provides a systematic approach to generate the initial states for the backwards integration, It automates what is otherwise a manual blind-search process. Furthermore, it discriminates amongst the uncountable families of trajectories going from the Earth to the halo orbit to provide an optimal transfer.

Another application, mentioned earlier, is the use of the heteroclinic transfer between lissajous orbits for the SU Mission. Prior to this approach, the intensive search for the return trajectory yielded next to nothing. Once this connection was made, the heteroclinic transfer method quickly produced an end-to-end trajectory which returned the spacecraft to a difficult site on the day side. Conventional methods used by another team of orbit designers produced trajectories which required more dells-V and were still unable to reach the Utah site required.

The PSI, FIRE, and SU case studies, as well as the work dorm by the European groups, clearly indicate the central role dynamical systems methods play in libration point trajectory design. It is an irony of history that the discipline giving birth to dynamical systems should now be one of the last to consider its application.

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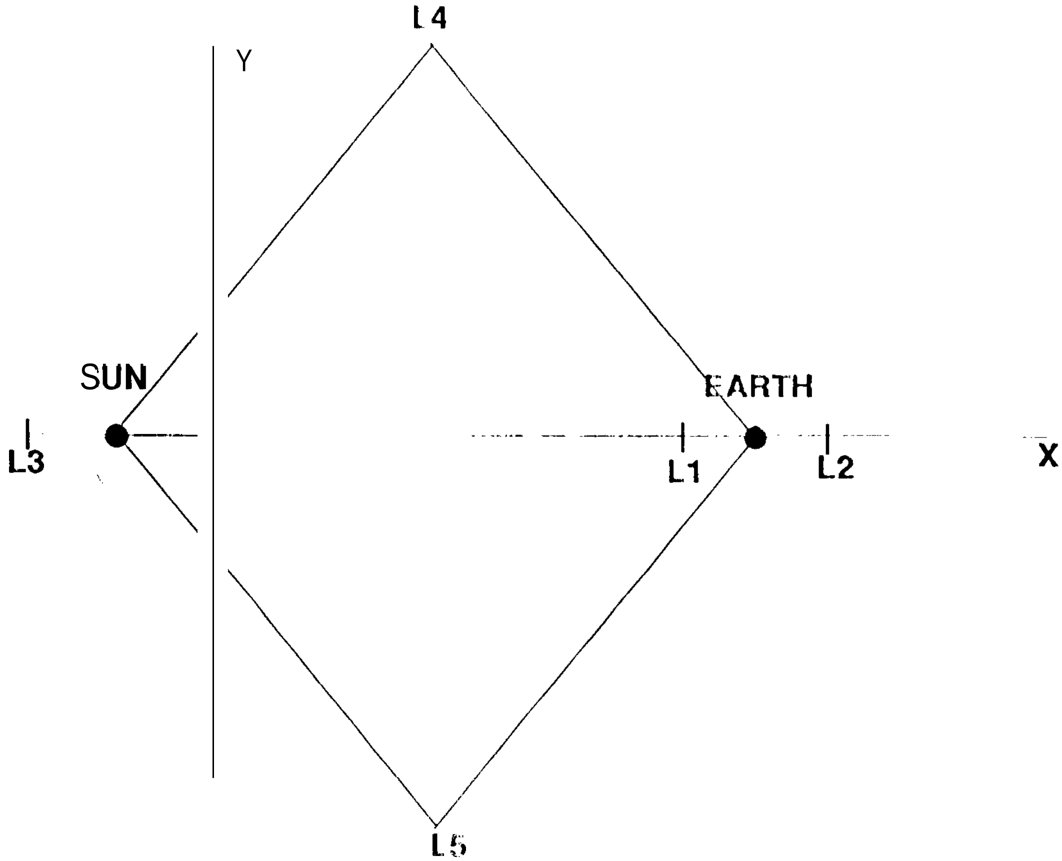

Figure 1. Sun-Earth System in Rotating Coordinate System

Figure 2. PSI/FIRE Orbit XY, XZ Projections

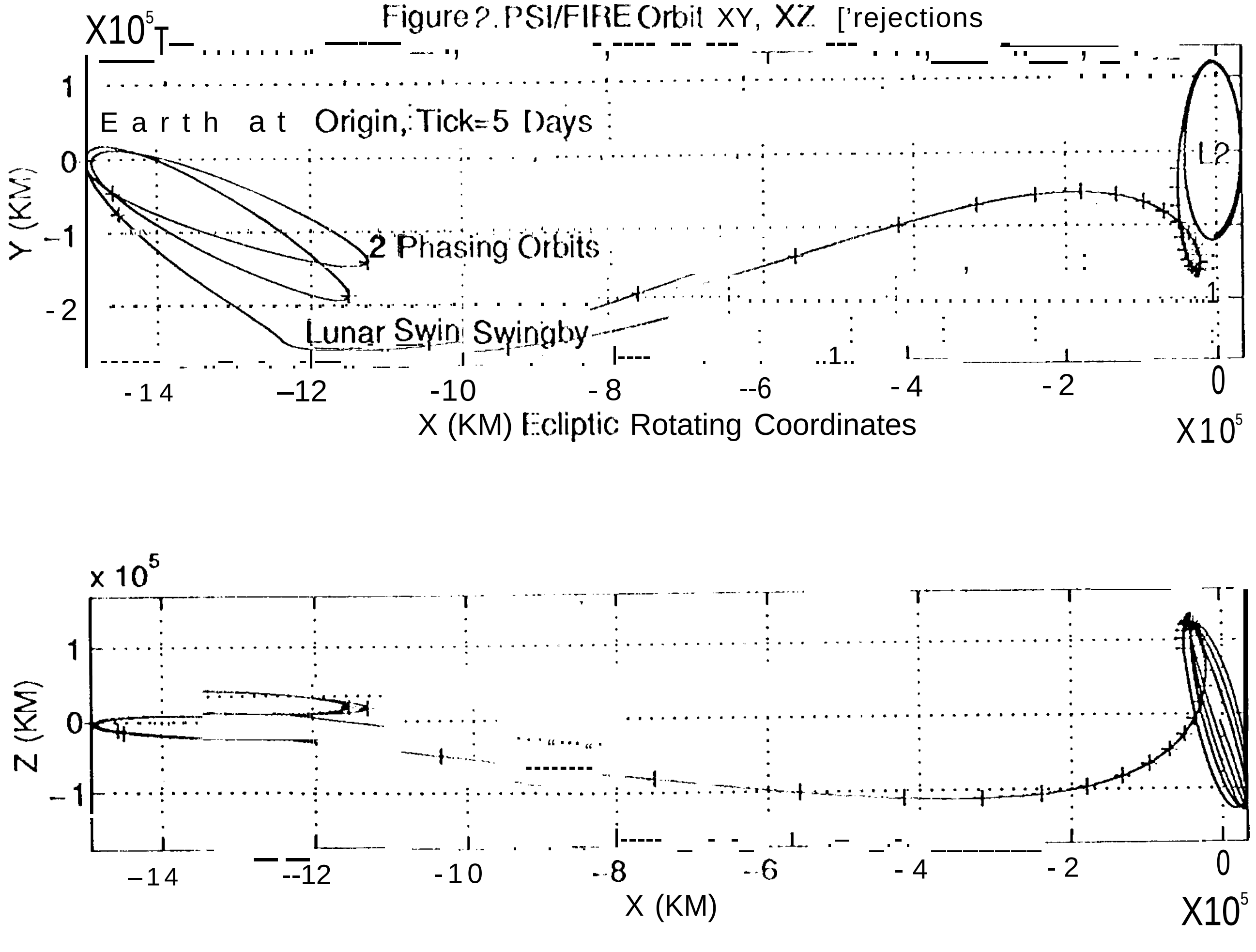

Fig.3 Suess-Urey Trajectory